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Scheme for the generation of entangled atomic state in cavity QED

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Abstract. We propose a scheme to generate the entangled state of two Λ-type three-level atoms trapped in a cavity. The atoms are initially prepared in their excited state and the cavity in vacuum state. Each atom has two possibilities to deexcite to one of the ground states. If two different polarized photons are detected subsequently, it is sure that both atoms are in different ground states. But which atom is in which ground state cannot be determined, the atoms are thus prepared in a superposition of two ground states, i.e., an entangled state. In comparison with the proposal of Hong and Lee [Phys. Rev. Lett. **89**, 237901 (2002)], the requirement of a single polarized photon source can be avoided in our scheme.

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Entanglement is considered to be one of the most striking features of quantum mechanics. It has come to be seen as a useful resource in achieving tasks of quantum communication and quantum computation [1]. Various quantum systems have been suggested as possible candidates for engineering of quantum entanglement. Among them the cavity-quantum-electrodynamics (CQED) systems are always paid more attention. In CQED the cold and localized atoms are not only the source of local entanglement, but also well suited for storing quantum information in long-lived internal states. Photons are the natural source for fast and reliable transport of quantum information over long distances. The resonant atom-cavity interaction, resulting in an energy exchange between the atom and the field, provides a direct mechanism to entangle the atomic and the cavity states. Recently numerous proposals in CQED have been made for entangling atoms in a single cavity [2–9] and atoms in two or more cavities [10–20]. Experimentally, two- and three-particle entangled states have been realized in CQED [21].

In a recent contribution [8] Hong and Lee proposed a simple scheme to entangle two identical Λ-type three-level atoms trapped in a single cavity, the atomic level structure is shown in Figure 1. Such an atomic level structure can be achieved using Zeeman sublevels [22]. The transitions $|e\rangle \leftrightarrow |g_L\rangle$ and $|e\rangle \leftrightarrow |g_R\rangle$ are strongly coupled, respectively, to left- and right-circularly polarized cavity modes.

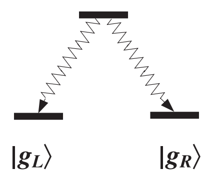

Fig. 1. Atomic level structure.

The atoms are prepared in the same ground state $|g_L\rangle$ and cavity in a two-mode vacuum state $|0_L, 0_R\rangle$. In the experiment, a left-circularly polarized photon is injected into the cavity. If a right-circularly polarized photon is detected afterwards, one can make sure that one of the atoms emits a right-circularly polarized photon and finally remains in the ground state $|g_R\rangle$. But because one cannot determine which atom is in $|g_R\rangle$ and which one in $|g_L\rangle$, the final state of the two atoms is an entangled state.

The main difficulty of their scheme is a requirement of a single photon source. Although there have been several schemes presented to realize such sources [23], it is far from practicality. To avoid such a requirement, in this paper we present a scheme to improve the proposal of Hong and Lee. We replace the single photon source by two laser pulses to prepare the two atoms in their excited state $|e\rangle$. Each excited atom has two possible transitions $|e\rangle \rightarrow |g_L\rangle$ or $|e\rangle \rightarrow |g_R\rangle$, corresponding to an emission of a left- or a right-circularly polarized photon. If two photons with

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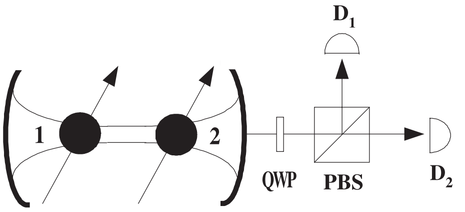

Fig. 2. Experimental set-up. Two atoms 1 and 2 trapped in a cavity are initially prepared to their excited state $|e\rangle$ by laser pulses. The photons leaking out from the cavity are detected by photodetectors D_1 and D_2 after transmitting quarter wave plates (QWP) and a polarizing beam splitter (PBS).

different polarizations are detected subsequently, the two atoms are sure to be in the different ground states, one in state $|g_L\rangle$ and the other in state $|g_R\rangle$. But which atom is in which ground state cannot be determined with certainty, the two atoms is thus entangled.

In our model the two atoms, whose level structure is depicted in Figure 1, are trapped in a cavity and each is individually addressed with laser light. The distance between the atoms is supposed to be much larger than an optical wavelength, so that dipole-dipole interactions can be neglected. In addition, this requirement allows us to assume that each atom can be individually addressed with laser light. The photons leaking out from the cavity first transmit a quarter wave plate (QWP), then are focused on a polarizing beam splitter (PBS), and finally detected by two photodetectors D_1 and D_2 [see Fig. 2]. We assume that the cavity is one sided so that the only leakage of photons occurs through the side facing QWP.

In order to illustrate our approach explicitly, let us first examine the resonant interaction between the Λ-type three-level atoms and the cavity modes in the case of an ideal cavity. The cavity is supposed to possess two modes with left- and right-circular polarization respectively. For the sake of simplicity, we also neglect the atomic spontaneous emission to other modes. The interaction Hamiltonian takes the following form

$$H_I = \hbar \sum_{j=1,2} \sum_{k=L,R} \lambda_k \left(a_k |e\rangle_{jj} \langle g_k| + a_k^\dagger |g_k\rangle_{jj} \langle e| \right), \quad (1)$$

where the subscripts $k = L, R$ present the left- and right-circularly polarized cavity modes respectively. $a_k^\dagger$ and a_k are the creation and annihilation operators of photons of the k mode, λ_k is the coupling constant between the k mode and the atom. We suppose λ_k to be real, and for the sake of generality we allow the coupling between the atom and the cavity modes to be different, i.e., $\lambda_L \neq \lambda_R$. If the atoms are prepared initially in their excited state $|e\rangle$ by laser pulses and cavity modes is in their vacuum states $|0_L, 0_R\rangle$, where $|0_L, 0_R\rangle \equiv |0_L\rangle \otimes |0_R\rangle$, one can find the evolution of the state of the atom-field system after the interacting time t,

$$\begin{aligned} |\Psi(t)\rangle = {} & C_1 |e, e\rangle_{1,2} |0_L, 0_R\rangle + C_2 \left(|g_L, e\rangle_{1,2} \right. \\ & \left. + |e, g_L\rangle_{1,2} \right) |1_L, 0_R\rangle + C_3 \left(|g_R, e\rangle_{1,2} + |e, g_R\rangle_{1,2} \right) |0_L, 1_R\rangle \\ & + C_4 |g_L, g_L\rangle_{1,2} |2_L, 0_R\rangle + C_5 |g_R, g_R\rangle_{1,2} |0_L, 2_R\rangle \\ & + C_6 \left(|g_L, g_R\rangle_{1,2} + |g_R, g_L\rangle_{1,2} \right) |1_L, 1_R\rangle, \end{aligned}$$

where

see equations (2–9) below.

$$C_1 = 1 + 4 \left\{ \frac{\left[(r_1 + a) \lambda_R - c\lambda_L \right]^2}{r_1 \left[c^2 + (r_1 + a)^2 \right]} \sin^2 \left(\frac{1}{2} \Omega_1 t \right) + \frac{\left[(r_2 + b) \lambda_L - c\lambda_R \right]^2}{r_2 \left[c^2 + (r_2 + b)^2 \right]} \sin^2 \left(\frac{1}{2} \Omega_2 t \right) \right\}, \quad (2)$$

$$C_2 = i \left\{ \frac{c \left[(r_1 + a) \lambda_R - c\lambda_L \right]}{\Omega_1 \left[c^2 + (r_1 + a)^2 \right]} \sin \Omega_1 t - \frac{(r_2 + b) \left[(r_2 + b) \lambda_L - c\lambda_R \right]}{\Omega_2 \left[c^2 + (r_2 + b)^2 \right]} \sin \Omega_2 t \right\}, \quad (3)$$

$$C_3 = i \left\{ - \frac{(r_1 + a) \left[(r_1 + a) \lambda_R - c\lambda_L \right]}{\Omega_1 \left[c^2 + (r_1 + a)^2 \right]} \sin \Omega_1 t + \frac{c \left[(r_2 + b) \lambda_L - c\lambda_R \right]}{\Omega_2 \left[c^2 + (r_2 + b)^2 \right]} \sin \Omega_2 t \right\}, \quad (4)$$

$$C_4 = 4\sqrt{2} \lambda_L \left\{ - \frac{c \left[(r_1 + a) \lambda_R - c\lambda_L \right]}{r_1 \left[c^2 + (r_1 + a)^2 \right]} \sin^2 \left(\frac{1}{2} \Omega_1 t \right) + \frac{(r_2 + b) \left[(r_2 + b) \lambda_L - c\lambda_R \right]}{r_2 \left[c^2 + (r_2 + b)^2 \right]} \sin^2 \left(\frac{1}{2} \Omega_2 t \right) \right\}, \quad (5)$$

$$C_5 = 4\sqrt{2} \lambda_R \left\{ \frac{(r_1 + a) \left[(r_1 + a) \lambda_R - c\lambda_L \right]}{r_1 \left[c^2 + (r_1 + a)^2 \right]} \sin^2 \left(\frac{1}{2} \Omega_1 t \right) - \frac{c \left[(r_2 + b) \lambda_L - c\lambda_R \right]}{r_2 \left[c^2 + (r_2 + b)^2 \right]} \sin^2 \left(\frac{1}{2} \Omega_2 t \right) \right\}, \quad (6)$$

$$C_6 = 2 \left\{ \frac{\left[(r_1 + a) \lambda_L - c\lambda_R \right] \left[(r_1 + a) \lambda_R - c\lambda_L \right]}{r_1 \left[c^2 + (r_1 + a)^2 \right]} \sin^2 \left(\frac{1}{2} \Omega_1 t \right) + \frac{\left[(r_2 + b) \lambda_R - c\lambda_L \right] \left[(r_2 + b) \lambda_L - c\lambda_R \right]}{r_2 \left[c^2 + (r_2 + b)^2 \right]} \sin^2 \left(\frac{1}{2} \Omega_2 t \right) \right\}, \quad (7)$$

$$a = 6\lambda_L^2 + \lambda_R^2, \quad b = 6\lambda_R^2 + \lambda_L^2, \quad c = 3\lambda_L \lambda_R, \quad (8)$$

$$r_{1,2} = -\frac{1}{2} (a + b) \pm \frac{1}{2} \sqrt{(a - b)^2 + 4c^2}, \quad \Omega_{1,2} = \sqrt{-r_{1,2}} \quad (9)$$

If we find both detectors D_1 and D_2 click, the state (2) will project into a maximally entangled state

$$\frac{1}{\sqrt{2}}\left(|g_L\rangle_1|g_R\rangle_2 + |g_R\rangle_1|g_L\rangle_2\right) \tag{10}$$

with the probability $P = |C_6|^2$. Otherwise, if none of the photodetectors clicks or only one of them clicks during the waiting time, we fail to generate the desired entangled state, and should repeat the process again until we find both photodetectors D_1 and D_2 click. It is interesting to note that the values of coupling constants λ_L and λ_R do not affect the fidelity of the final entangled states (10).

The scheme given here is similar to that of Hong and Lee [8]. But physically the two schemes are different. In the scheme of Hong and Lee the atomic entanglement is generated in photon absorption process, while in our scheme it is generated in photon emission process. In addition, the requirement of the single photon source can be avoided.

In the above discussion we have neglected the influence of the cavity decay and the atomic spontaneous emission to other modes on our model. Now let us take into account the influence of the cavity decay first. Considering the cavity decay and the photon observation, the quantum trajectory theory [24,25] is a very suitable method. The basic idea of this theory is that the evolution of the quantum system under continuous detection, conditional to observing a particular trajectory of counts, can be described by a pure state wave function $|\Psi_c(t)\rangle$ which is governed by a non-Hermitian effective Hamiltonian H_{eff},

$$H_{\text{eff}} = H_I - i\hbar k \sum_{k=L,R} a_k^\dagger a_k, \tag{11}$$

where the first term H_I is exactly equation (1) and other terms result from the cavity decay into the environment. During the time interval when no photon is detected, the wave function evolves according to this non-Hermitian effective Hamiltonian (11). The detection of photons is accompanied by the wave function collapse $|\Psi_c(t)\rangle \to \hat{C}|\Psi_c(t)\rangle$, and the probability density for such a detection to occur at time t is $P = \langle\Psi_c(t)|\hat{C}^\dagger\hat{C}\ |\Psi_c(t)\rangle$.

It it obvious that in the case that no photon is detected the wave function $|\Psi_c(t)\rangle$ takes the similar form to the wave function $|\Psi(t)\rangle$ which is described by equation (2). The difference between $|\Psi_c(t)\rangle$ and $|\Psi(t)\rangle$ consists in their coefficients C_i's. We assume the corresponding coefficients of $|\Psi_c(t)\rangle$ are C_i'. Substituting $|\Psi_c(t)\rangle$ into the Shrödinger equation which is governed by H_{eff}, we can obtain the following differential equations about C_i',

$$\dot{C}_1' = -2i\lambda_L C_2' - 2i\lambda_R C_3', \tag{12}$$

$$\dot{C}_2' = -i\lambda_L C_1' - \sqrt{2}i\lambda_L C_4' - i\lambda_R C_6' - kC_2', \tag{13}$$

$$\dot{C}_3' = -i\lambda_R C_1' - \sqrt{2}i\lambda_R C_5' - i\lambda_L C_6' - kC_3', \tag{14}$$

$$\dot{C}_4' = -2\sqrt{2}i\lambda_L C_2' - 2kC_4', \tag{15}$$

$$\dot{C}_5' = -2\sqrt{2}i\lambda_R C_3' - 2kC_5', \tag{16}$$

$$\dot{C}_6' = -i\lambda_R C_2' - i\lambda_L C_5' - 2kC_6'. \tag{17}$$

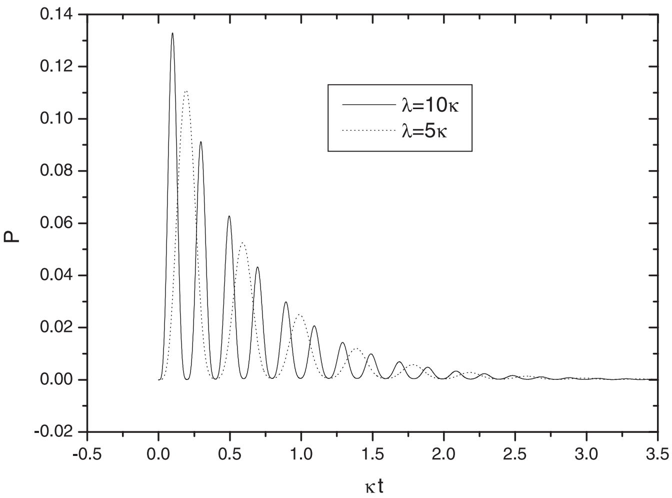

Fig. 3. Probability P of detecting two photons with perpendicular polarizations as a function of kt with different values of λ.

For the sake of simplicity, in the rest of this paper we suppose the coupling constants between the atom and the cavity modes are equal, i.e., $\lambda_L = \lambda_R = \lambda$. In such a case $C_2' = C_3'$, $C_4' = C_5'$. With the help of Laplace transformation we can work out all the coefficients C_i''s. Here we only write down the expression of C_6',

$$C_6' = \frac{2}{3}\frac{\lambda^2}{s_1^3 - s_2^3}\Bigg\{(s_1 - s_2)\exp[(s_1+s_2)\,t] - \left[(s_1-s_2)\cos(dt) + \sqrt{3}\,(s_1+s_2)\sin(dt)\right] \times \exp\left[-\frac{1}{2}(s_1+s_2)\,t\right]\Bigg\}\exp(-\kappa t), \tag{18}$$

where

$$s_1 = \sqrt[3]{\kappa\lambda^2 + \sqrt{\kappa^2\lambda^4 + \frac{1}{27}\left(10\lambda^2 - \kappa^2\right)^3}}, \tag{19}$$

$$s_2 = \sqrt[3]{\kappa\lambda^2 - \sqrt{\kappa^2\lambda^4 + \frac{1}{27}\left(10\lambda^2 - \kappa^2\right)^3}}, \tag{20}$$

$$d = \frac{\sqrt{3}}{2}\left(s_1 - s_2\right). \tag{21}$$

If both photodetectors D_1 and D_2 click at time t, two photons with perpendicular polarizations are annihilated with $\hat{C}$ which takes a form $\hat{C} = (a_{LA} + a_{LB})(a_{RA} + a_{RB})$ [16]. In this case the wave function $|\Psi_c(t)\rangle$ collapses into the desired entangled state (10) with the probability density $P = |C_6'(t)|^2$. In Figure 3 we plot the probability P as a function of kt with the different values of λ. We find that, in the good cavity case ($\lambda > k$), the waiting time for the effective detection can be chosen to be a few times of cavity lifetime $1/k$.

Finally let us take into account of the influence of the atomic spontaneous emissions and other elements. On the one hand, the photons from the spontaneous emissions to the free modes (rather than the cavity modes) run in random directions and can not be detected by photodetectors D_1 and D_2, therefore the efficiency of successful preparation of the desired entangled state is reduced. The quantity of such a negative influence on the successful probability density P is obviously related to γ, the rate of the spontaneous emissions to the free modes. And the total successful probability density is thus reduced to P–γ. However, this will not affect the fidelity of the resultant entangled state.

On the other hand, because the time when an atomic spontaneous emission occurs is stochastic and uncontrollable and the photon decays from the cavity are also random, one cannot assure both atoms deexcite at the same time. Correspondingly we might obtain these detecting results that both photodetectors D_1 and D_2 click at different time. In this case, the sources of the detected photons may be identified according to the early or late occurrence of atomic recoil, the entanglement fidelity of the resultant entangled state is thus reduced. Otherwise, considering the experimental conditions in existence, it is difficult to keep atoms in the cavity exactly at the same positions from one experiment to another, and thus the differences in the phases of photons related with the atom position uncertainty will also reduce the fidelity of the resultant entangled state. So it is a very important problem in the practical experiment to keep the atoms in the traps in the Lamb-Dicke regime where no recoil is imparted upon them.

In summary we have proposed a scheme to generate the entangled state of two Λ-type three-level atoms trapped in a cavity. In comparison with the proposal of Hong and Lee, the requirement of a single polarized photon source can be avoided in our scheme.

We note that the several schemes for the generation of entangled states of two or more atoms in a single cavity, such as the ones in [3,9] which also do not require the nonclassical light sources, have been proposed recently. The difference between our scheme and the one in [3] consists in the photon detecting result to achieve the atomic entanglement. In order to obtain the desired entangled states of atoms, the detecting result should be both photon detectors click in our scheme, while in the scheme of [3] the result for detecting photons is required to be null. The difference between our scheme and [9] is that the latter requires the initial atomic states be prepared in a superposition state, this obviously increases the difficulty in the practical experiments. We also note that after the submission of this work similar contributions have been published [26].

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